Undrained behaviour of sand with fines and Critical State Soil Mechanics

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Abstract: The Steady State Line (SSL) for sand with fines moves downward with increasing fines (particle diameter < 75 μm) content. This means that sand with fines may have an indefinite number of SSLs in *e*–log(*p*′) space, where *e* and *p*′ are void ratio and mean effective stress at steady state respectively. Thus, one cannot have a single set of parameters within the critical state soil mechanics (CSSM). To overcome this issue, equivalent granular void ratio, *e** instead of void ratio, *e* is proposed by many researchers. However, different approaches can be found in the literature to achieve *e**. This paper presents a comparison among those different approaches by using five published databases. Then, the most general approach to obtain *e** is indentified and used to synthesize a new experimental dataset for sand with up to 30% fines content. A single trend for steady state data points, referred to as Equivalent Granular Steady State Line (EG-SSL), is achieved in *e**–log(*p*′) space irrespective of fines content. The effective stress path and stress-strain behaviour for clean sand and sand with fines under undrained condition are consistent when the EG-SSL is used as the reference line within the CSSM framework.

Key words: Steady state, sand, fines, void ratio, equivalent granular void ratio

1 INTRODUCTION

Early studies on liquefaction were mainly concentrated on clean sands (particle diameter > 75 μm) although the presence of fines (particle diameter ≤ 75 μm) in sands is not uncommon. Since the 1960's, it has been understood that the presence of fines in some manner affects the resistance to liquefaction of sands. But only few systematic studies have been carried out on sandy soils under the critical state soil mechanics (CSSM) during last 20 years (Zlatovic and Ishihara, 1995, Cubrinovski and Ishihara, 2000, Thevanayagam, *et al.*, 2002, Yang, *et al.*, 2006, Bobei, *et al.*, 2009, Rees, 2010). These studies showed steady state data points move downward in *e*–log(*p*′) space up to a certain limiting fines content, followed by an upward move with increase in fines content (as shown in Figure 1), where *e* and *p*′ are void ratio and mean effective stress at steady state respectively. This apparently means that there are an indefinite number of steady state lines (SSLs) for sand with fines in *e*–log(*p*′) space. This introduces a big challenge to the geotechnical engineers in representing the behaviour of sand with fines with a single set of parameters under the CSSM framework.

However, many researchers have reported from the experimental studies that void ratio, *e* may not be a good state index for sandy soils and equivalent granular void ratio, *e** instead of void ratio, *e* should be used so that the steady state data points can be coalesced to achieve a single relationship in *e**–log(*p*′) space for sand with different fines content (Thevanayagam, 1999, Chiu and Fu, 2008). The single trend line of steady state data points is called the equivalent granular steady state line, EG-SSL. The theory behind the adoption of *e** is that if fines particles are sufficiently small, most of the fines do not play an active role in the force chain of sand particles, and only a fraction of fines may be active in the force chain of sand particles as illustrated in Figure 2. Thus, by considering the inactive fraction of fines as void, Thevanayagam *et al.* (2002) define *e** as-

$$e^* = \frac{e + (1-b)\,f_c}{1-(1-b)\,f_c} \qquad (1)$$

where, *b* is the fraction of fines that are active in the sand force chain and f_c is fines content (in decimal). The above equation for *e** is only valid for 'fines-in-sand' model i.e. up to a threshold fines content, f_{thre}. For higher fines content, the movement of steady state data points changes direction from downward to upward in *e*–log(*p*′) space. However, the main challenge of using *e** is to obtain a physically reasonable *b* value. Most of the studies found in literature used either a back analyzed or an assumed *b* value. In 2005, a correlation between back analyzed b and particle grading properties was proposed by Kanagalingam and Thevanayagm (2005), however the ability of the correlation in predicting/estimating e^* has not been independently evaluated. Recently, Rahman et al. (2008, 2009) proposed a prediction formula for b from grading properties of parent sand and fines. To the best of author's knowledge, this is the only prediction technique for b and thus, referred to as prediction approach hereafter. The objective of this study is to evaluate these approaches side by side using large published databases. Then the prediction approach is used to evaluate e^* and thus, the EG-SSL for a new experimental study. Further e^* and the EG-SSL, obtained by prediction approach, are used to predict undrained behaviour within a single CSSM framework irrespective of fines content.

2 COMPARISON AMONG DIFFERENT APPROACHES

Three different approaches have been found for getting b value; Back Analysis (BA), Thevanayagam and Co-workers Correlation (TCC) and Prediction Approaches (PA).

A short description of these approaches can be found in appendix-A. The first approach is purely based on back analysis. The second approach is also based on back analysis; however it gives a correlation between b and soil grading properties. These two approaches assume a constant b value irrespective of fines content although this is contradictory to 'fines-in-sand' model originally proposed by Thevanayagam et al. (2002). The third approach is a true prediction approach where inputs are soil grading properties and fines content. In this case, b depends on fines content. Five databases were collected from the literature to evaluate these approaches. The grading properties of those materials are given in Table 1.

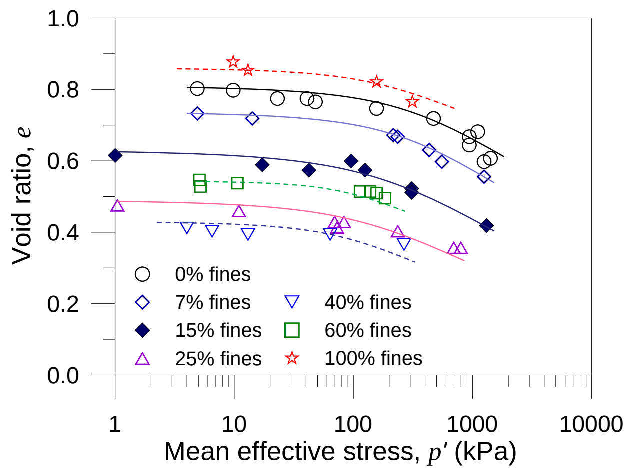

Figure 1: The location of steady state data points for sand with fines, data after Thevanayagam et al. (2002).

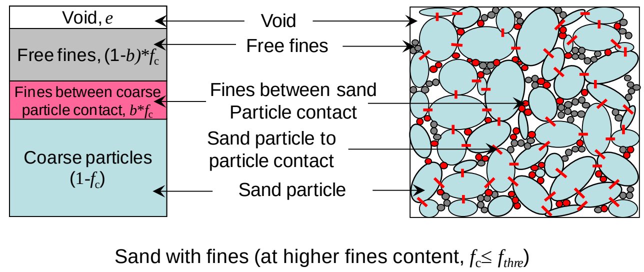

Figure 2: Schematic diagram of equivalent granular void ratio concept.

The b was obtained by three different approaches as outlines in appendix-A. Then, e^* and the EG-SSL for these databases were calculated. The scatter of the steady state data points around the best fit EG-SSL in e^*-log(p') space were used to evaluate the performance for each of these methods. A statistical measure, root-mean-square-deviation (RMSD) was used for this purpose. The best performance in getting the EG-SSL was obtained for BA. This is as-expected because b was selected by trial and error-

Table 1: Different approaches of getting b and their performances with five published databases.

References	Sand				Fines			f_{thre}	RMSD for different approaches		
	Name	D_{50}	D_{10}	U_S	Name	d_{50}	U_F		Back analysis	TCC*	Prediction approach
1	OS00	0.250	0.160	1.69	Silica	0.010	7.50	0.36	0.019	0.019	0.027
2	Mai Liao	0.123	0.080	1.75	Mai Liao	0.044	2.79	0.41	0.027	0.100	0.028
3	Alluvium	0.778	0.209	5.63	Alluvium	0.038	5.43	0.30	0.014	0.026	0.015
4	Hokksund	0.440	0.225	2.25	Chengbei	0.032	2.32	0.30	0.033	0.110^	0.032
5	Toyoura	0.170	0.116	1.61	Toyoura	0.010	6.08	0.33	0.022	0.038	0.051#

[1]Thevanayagam et al. (2002), [2]Haung et al. (2004), [3]Ni et al. (2004), [4]Yang et al. (2006), [5]Zlatovic and Ishihara(1995)
#much smaller RMSD can be achieved without f_c = 0.30; *Thevanayagam and coworker's correlation (TCC); ^$U_S U_F^2$/Rd<1 beyond correlation and b = 0.15 (the lowest value taken from the correlation).

to give an EG-SSL with least scatter. The worst performance was obtained for TCC as presented in Table 1. The square of uniformity coefficient of fines, U_F was used as the numerator in the grading factor in TCC approach. Thus, the uncertainty associated with U_F significantly contributed to the scatter in the TCC method. However, the overall performance for prediction approach is very closed to BA and can be consider as good as BA. Thus, the prediction approach is used to synthesize data for the experimental study presented in this paper.

3 EXPERIMENTATION

A series of undrained triaxial tests on sand-fines mixtures was conducted to cover a wide range of testing conditions: i) mean effective stresses at start of shearing, p'_0, ranging from 100kPa to 1300kPa, ii) three different fines contents (below f_{thre}) and iii) void ratios at the start of shearing, e_0, ranging from 0.455 to 0.892. The details of the testing program can be found in Rahman (2009). However, only few relevant test results are presented in this paper.

3.1 Testing materials

The host sand, referred to as Sydney sand, was a clean uniform size quartz sand (SP) and the fines was reconstituted from 2/3 of the Majura River bank deposits and 1/3 of commercial kaolin. The fines are referred to as MII fines. The details of the sand and fines properties are given in Table 2 and the grain size distributions are shown in Figure 3.

3.2 Experimental setup

The experimental investigation was based on triaxial testing with fully automated data logging facilities. A schematic diagram for the testing facilities and data acquisition system is given in Figure 4. Axial load was measured with an internal load cell. The axial deformation was measured by two independent means: a pair of internal LVDTs mounted directly across the top platen and an external LVDT. The former was used in the early stage of shearing whereas the latter was used at large deformation. Cell pressure was controlled by a large capacity Digital Pressure Volume Controllers (DPVC). The pore pressure line was connected to a small capacity DPVC for controlling back pressure (and measuring the volume change at the consolidation stage) and for imposing an undrained condition with measurement of the resultant pore pressure response. Two pressure transducers were also used to verify pore pressure equilibrium.

Table 2: Properties of the testing materials.

Parameters	Sand	MII fines
e_{max}	0.855	---
e_{min}	0.565	---
Gs	2.63	2.46
U_C	1.26	21.56
D_{10}	0.225	---
d_{50}	---	0.0055
PI	---	27

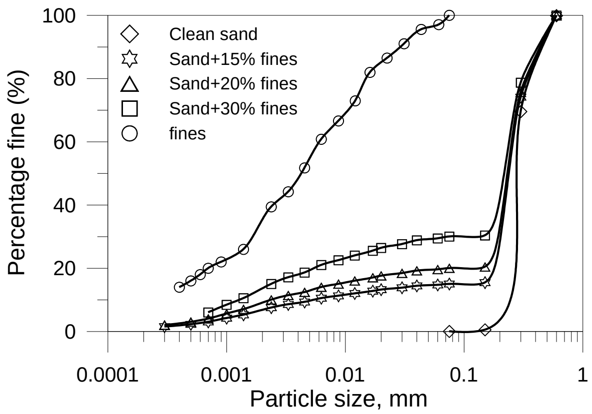

Figure 3: Grain size distribution of testing materials.

3.3 Sample preparation

A modified moist tamping method was used for sample preparation to ensure uniformity of the specimen. To accurately control the void ratio, a total of 10 layers of predetermined quantities of moist soil were worked into a prescribed thickness as detailed in Rahman (2009). Enlarged end platens with free ends, as described by Lo et al. (1989), was used to minimize end restraint. Saturation of the specimen was accomplished in two stages: (i) CO_2 percolation followed by vacuum flushing under a small head, and (ii) Back pressure application. A Skempton B-value of at least 0.98 was achieved in all the tests. Uniform deformation of the sample was obtained up to the end of shearing to a large strain as shown in Figure 5.

3.4 Measurement of void ratio

A good void ratio measurement is needed for examining the behaviour of a soil using the CSSM framework. For a loose sandy specimen, the overall specimen dimensions will change during saturation. During the vacuum flushing stage, the specimen dimensions were directly measured and therefore, the void ratio at the end of vacuum flushing was accurately determined. The change in specimen volume, thus void ratio, after back-pressure saturation was also accurately tracked by the DVPC.

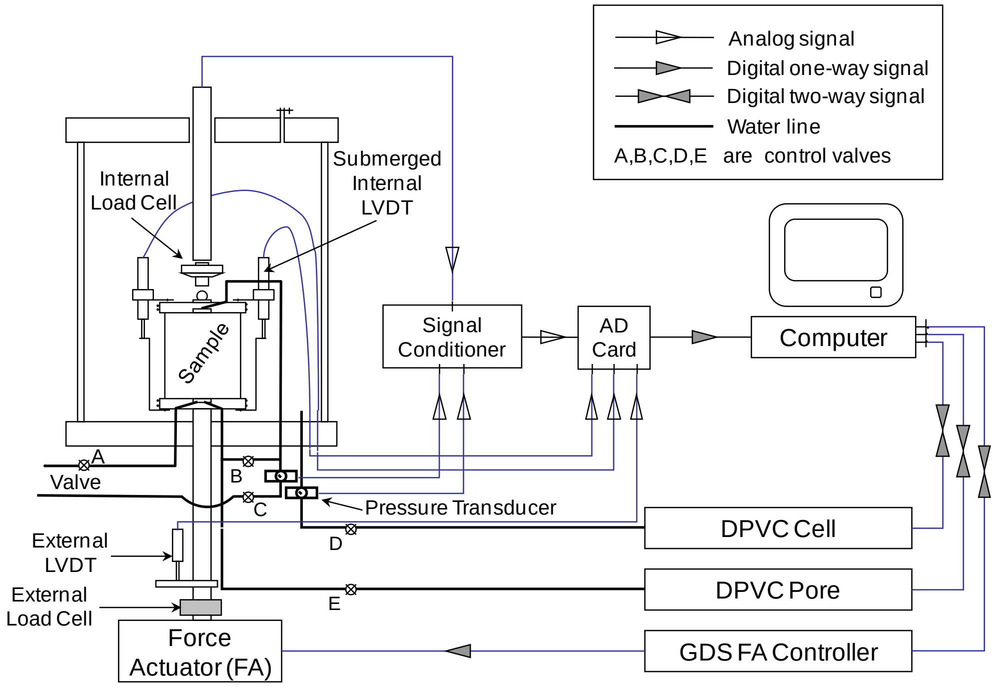

Figure 4: Schematic diagram of triaxial machine and data acquisition system.

Although the change in specimen diameter could not be measured during back-pressure saturation, the change in axial strain, $\delta\varepsilon_1$, was accurately monitored by the internal LVDTs at all stages. Both Bobei (2004) and Rahman (2009) neglected the small change in specimen diameter during back-pressure saturation. The resultant error is small because the specimen was close to saturation at end of vacuum flushing. In this paper, the change in radial dimensions was inferred from $\delta\varepsilon_3 \approx 2\delta\varepsilon_1$, where $\delta\varepsilon_3$ denotes change in radial strain during back pressure saturation.

Figure 5: Uniform deformation of the specimen during shearing up to a large strain of $\varepsilon_1 > 30\%$.

4 RESULTS AND DISCUSSIONS

A total of 28 tests have been done for Sydney sand with up to 30% fines content. Most of the specimens reached steady state at the end of the test. However, A few specimens approached but did not reach steady state. In such a case, the steady state was obtained by the extrapolation method proposed by Murthy et al. (2007). The effective stress paths, stress-strain behaviour and steady state for sand with 15% fines content at different confining stresses are presented in Figure 6.

4.1 *Steady state behaviour*

The steady state data points for clean sand and sand with up to 30% fines are presented in Figure 7a-b. The downward movement of steady state data points for sand with fines in e–log(p') space is consistent with others (Zlatovic and Ishihara, 1995, Thevanayagam, *et al.*, 2002, Yang, *et al.*, 2006, Bobei, *et al.*, 2009). Thus, sand with fines behaviour cannot be analyzed with a single set of parameters under the CSSM. However, SSL for a particular fines content can be used to analyze behaviour for sand with that particular fines content. This requires extensive number of tests to define SSLs for every possible fines content. However, the steady state data points exhibited a single relationship in the e^*-log(p') space irrespective of f_c as shown in Figure 7b. It is pertinent to re-iterate that, unlike most of the earlier work, a single EG-SSL has not been assumed in the data analysis as e^* was obtained from the prediction approach. However, this EG-SSL is a curve which is consistent with many publications (Wang, *et al.*, 2002, Bobei and Lo, 2005, Rahman and Lo, 2007). It can be represented by the following power function as proposed by Wang et al. (2002).

$$e^{*} = e_{\lim} - \Lambda\left(\frac{p'}{p_a}\right)^{\xi} \qquad (2)$$

where p_a is 100kPa expressed in a unit consistent with that used in p'. The parameters of the power function are: $e_{\lim}$ = 0.920, Λ = 37.5*10^{-3}, and ξ= 0.60. The scatter of the data points around this power function is minimal (RMSD = 0.016), and further suggests the applicability of the prediction approach.

4.2 Overall undrained behaviour within a CSSM framework

In order to evaluate whether a single set of parameters under the CSSM can be used to predict the overall behaviour pattern of sand-fines mixtures for a wide range of $f_c \leq f_{thre}$, the initial states (i.e. just prior to shearing) of all the tests were plotted in e^*-log(p') space together with the EG-SSL in Figure 8. The data points located below the EG-SSL, all corresponded to non-flow behaviour (no-liquefaction) as indicated by the symbol "•". The data points located clearly above the EG-SSL corresponded to flow behaviour (as indicated by the "o" symbol). Several data points plotted close to the EG-SSL corresponded to limited-flow behaviour as indicated by the symbol "×". The above observation unambiguously showed that a single CSSM framework can be obtained to predict the overall behaviour pattern in undrained shearing. The next step is to examine the use of e^* and the EG-SSL in predicting undrained effective stress path (ESP) and stress-strain responses.

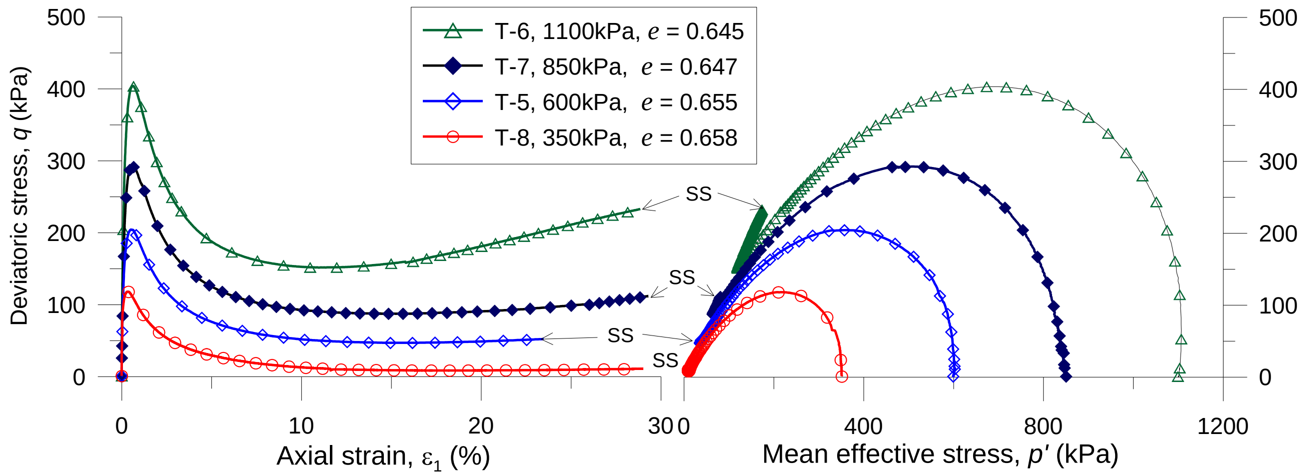

Figure 6: Effective stress paths, stress-strain behaviour and steady states for sand with 15% fines content at different confining stresses.

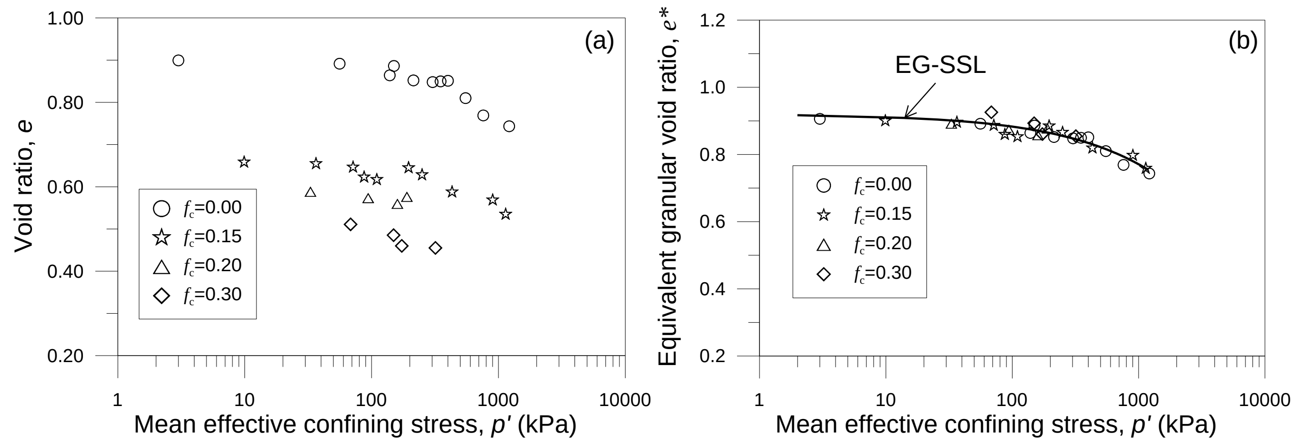

Figure 7: Steady state data points for sand with fines; (a) SSLs in e-log(p') space, (b) EG-SSL in e^*-log(p') space.

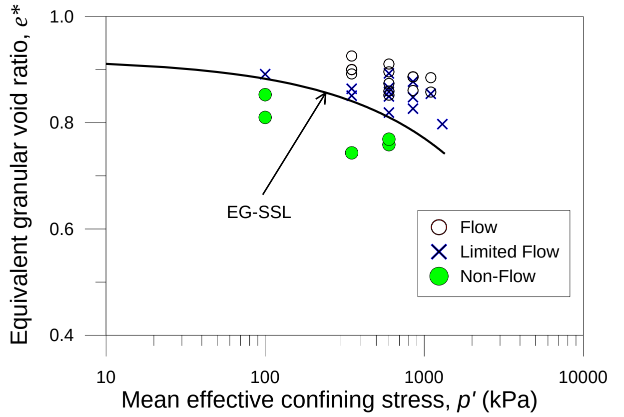

Figure 8: Initial states of all samples (sand with fines) relative to the EG-SSL.

4.3 *Effective stress paths and stress-strain responses within a CSSM framework*

This sub-section compares the ESP and q-ε_1 responses of three tests conducted on samples with different fines content and different initial states. The mean effective stress, p'_0 of these three tests were 600 kPa and thus, initial states of these tests in e^*-log(p') space depend solely on e^*. The equivalent granular void ratio at steady state, e^*_{SS} for p'_0=600kPa was 0.810 as obtained from equation (2). Thus, their ESPs and stress-strain behaviour should be correlated to e^* and e^*_{SS}. The ESPs are compared in Figure 9a. A correlation between the overall location of ESPs and their corresponding initial states can be observed. Test T-14 had f_c=0.20. It had e^* of 0.874 which is much higher than e^*_{SS} = 0.810. Thus, its ESP traced leftward (i.e. dp' < 0) during shearing, and plummeted toward the origin after attaining the peak deviator stress. Test NTC-01 had f_c=0.0 i.e. clean sand. It had e^* of 0.852, which is higher than the previous sample and closer to e^*_{SS}; it exhibited a transitional behaviour. Its ESP also traced leftward, but to a lesser extent compared to the test T-14. After attaining a peak q and then showing some reduction in q, the ESP reversed its direction, i.e. turning back towards dp' > 0 and climbed upwards. Test T-31 had f_c=0.15 and e^* lower than e^*_{SS}. Its ESP traced only slightly to the left and then turned right i.e. towards dp' > 0 and q increased continuously throughout the test. This test achieved the highest values, among all three tests, of q and p' at the end of the test.

The q-ε_q responses of the above three tests are shown in Figure 9b. Test T-14 (highest e^*) showed rapid strain softening towards the steady state after attaining its peak deviator stress. The q-ε_1 response of test NTC-01 (intermediate e^*) showed a gradual and less strain softening after attaining a peak deviator stress, and the behaviour soon reverted back to a gradual strain hardening. The q-ε_1 response of test T-31 (lowest e^*) showed a continuous strain hardening behaviour. Thus, the ESP and the deviatoric stress-strain responses of the above three tests for sand with fines can be correlated to e^* within a single CSSM framework despite the fines content of all three tests being different.

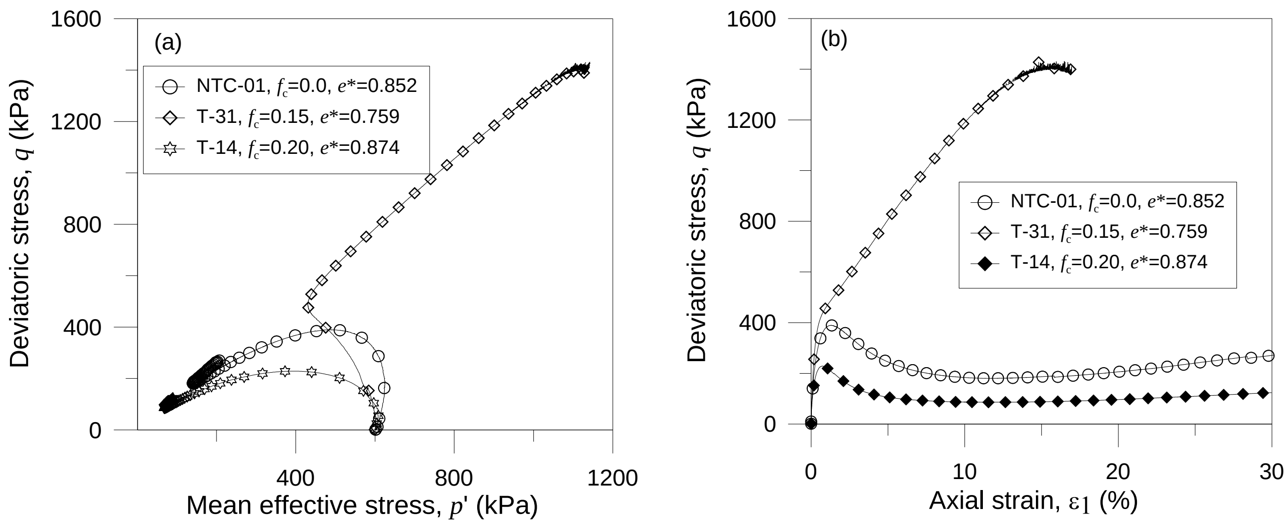

Figure 8: Undrained behaviour of sand with fines; (a) effective stress path and, (b) stress-strain responses.

5 CONCLUSIONS

The effects of fines on the undrained behaviour of sandy soils have been evaluated with a series of undrained triaxial tests for sand with fines and the tests results are synthesized in terms of equivalent granular void ratio, e^* instead of void ratio, e. Three different approaches have been adopted from the literature in converting e to e^*. This paper presents a comparison of these approaches and uses the most generalized prediction approach to synthesize the experimental data. The key findings can be summarized as follows:

- The prediction approach is the most generalized option of getting e^* from e using soil grading properties and fines content. The overall performance of the prediction approach is very close to back analyses for the databases used in this study.
- The EG-SSL can be used to predict overall behaviour of sand with fines up to the threshold fines content. The effective stress paths and the stress-strain behaviour for sand with fines are found to be consistent within a CSSM framework.
- The equivalent granular steady void ratio, e^* may be used to achieve a single set of parameters under the critical state soil mechanics (CSSM) framework for sand with different fines content.

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APPENDIX-A: Different approaches for the determination of *b*

Back Analysis (BA)

The back analysis is often based on the assumption that the steady state data points for sand-fines mixtures in the e^*-log(p') space can be described by a single relationship irrespective of f_c. Thus, a single *b* value was searched by trial and error until a single correlation was achieved for steady state data points for sand with fines. The main practical disadvantage of this approach is the requirement of large number of triaxial tests for different fines contents as input.

Thevanayagam and co-workers correlation (TCC)

Thevanayagam and co-workers (Kanagalingam and Thevanayagam, 2005) reported that back analyzed *b* depends on three factors: particle size ratio (Rd = D_{50}/d_{50}), uniformity coefficient of sand (U_S) and uniformity coefficient of fines (U_F). They also reported a correlation between *b* and $U_S U_F^2$/Rd as shown in Figure A1. However, the scatters in the data points are noticeable. The *b* value varies from 0.40 to 0.80 for the little variation of $U_S U_F^2$/Rd from 10 to 11 and *b* varies a little from 0.20 to 0.45 for large variation of $U_S U_F^2$/Rd from 3 to 10. It should be noted that *b* is also independent of f_c though the 'fines-in-sand' support that *b* is a function of f_c (Thevanayagam, *et al.*, 2002). Thus, the *b* value obtained in these cases may be an average value for a range of f_c.

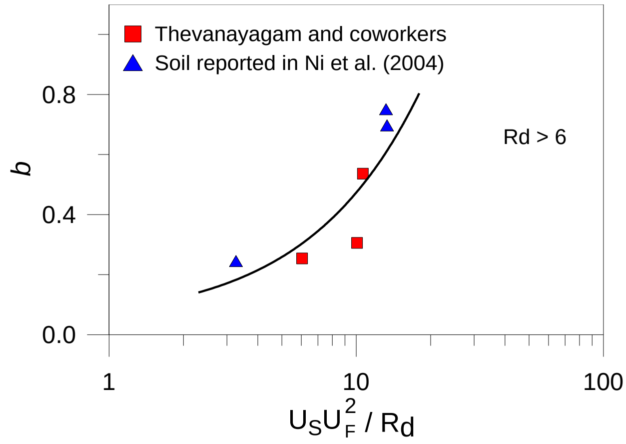

Figure A1: The correlation between *b* and $U_S U_F^2$/Rd, after Kanagalingam and Thevanayagam (2005).

Prediction Approach (PA)

Recently, Rahman et al. (2008, 2009), re-analysed the experimental data of McGeary (1961) on binary packing studies, and concluded that *b* is a function of both f_c and χ=D/d, where D is the size of sand and d is the size of fines. The functional relationship, $b = F(f_c, \chi)$, can be presented by the following equation-

$$b = \left[1 - \exp\left(-0.3\frac{(f_c / f_{thre})}{k}\right)\right] \times \left(r\frac{f_c}{f_{thre}}\right)^r \quad \text{(A1)}$$

where *r* = particle size ratio, d/D, $k = (1 - r^{0.25})$, f_{thre} is the threshold fines content. The fines content that defines the reversal in the movement of the location of steady state data points in *e*-log(p') space is called f_{thre}. It is noted that the concept of *b* is only applicable for $f_c < f_{thre}$. Since sand and fines are generally not single-size materials, D/d was generalized to D_{10}/d_{50} based on the argument in Ni et al. (2004), where the subscripts denote fractile passing. The f_{thre} is an input parameter for equation (A1) and it can be obtained from the experimental procedure outlined in Yang et al. (2006). However, it can be approximated as an average value of 0.30

or can be estimated by an empirical equation (A2) as presented in Rahman and Lo (2008) where enough information on steady state behaviour are not available.

$$f_{thre} = 0.40\left(\frac{1}{1+e^{0.50-0.13\chi}} + \frac{1}{\chi}\right) \quad \text{(A2)}$$

Thus, the equations (A1) and (A2) can be used to determine *b* without the need of back analyzing a substantial triaxial datasets covering a range of f_c.

NOTATIONS:

b	active fraction of fines in force structure
d	fines particle diameter
d_{50}	fines particle diameter at 50% finer
D	sand particle diameter
D_{10}	sand particle diameter at 10% finer
e	void ratio
e^*	equivalent granular void ratio
f_c	fines content in decimal
f_{thre}	threshold fines content in decimal
χ	particle size ratio, $\chi = D_{10}/d_{50}$
r	particle size ratio, $r = (1/\chi) = d_{50}/D_{10}$
p'	mean effective stress, $p' = (\sigma'_1+2\sigma'_3)/3$
p_a	reference stress, 100kPa
U_C	uniformity coefficient
U_S	uniformity coefficient of sand
U_F	uniformity coefficient of fines